A Unified Measure of Audio System Fidelity

A Thesis

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Dedication

To my wife, Jennifer, my mom and dad, and my sister without their love and support this would not be possible.

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This manuscript was a true team effort. From the beginning, my Major Professor, Dr. Terry E. Riemer, guided its creation from start to finish imparting his special insight and many years of expertise. I am also very thankful for the generosity of Biomedtronics, L.L.C. whose continuous support made this effort possible. I would also like to thank the Math Works, Inc. and their revolutionary software MATLAB® and Simulink® which assisted greatly in implementations of all of the models and theory in this manuscript.

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List of Abbreviations

A/D: analog to digital converter
AGC: automatic gain control
ANSI: American National Standards Institute
ANSI S3.22: hearing aid standard
dB: decibel
FET: field effect transistor
FDA: Food and Drug Administration
IMD: intermodulation distortion
NPTS: number of sample points
PSD: power spectral density
SPL: Sound Pressure Level
THD: total harmonic distortion

Abstract

A new technique to qualitatively measure distortion in dynamically controlled audio systems using non-stationary noise sequences is explored and compared to traditional methods based upon stationary test signals. This technique can easily be adapted to give a qualitative measure of distortion as a function of the perceived Sound Pressure Level (SPL).

Keywords: Total Harmonic Distortion Measurement, Coherence Distortion Measurement, Incoherence Distortion Measurement, Measurement of Audio System Non-linearity, Lawrence J. Tedesco, Jr., Lawrence J. Tedesco, Jim Tedesco, Dr. Terry E. Riemer, Dr. T. E. Riemer

Chapter 1
Background

The term distortion as applied to an audio signal is a qualitative measure of any perceived undesirable signal components as a consequence of the psychoacoustic processing of the human ear. Minimizing distortion is the most serious problem facing the audio engineer in designing any high fidelity sound reproduction system. To quote one of the most famous pioneers in audio engineering, “If it measures good and sounds bad, it is bad; if it measures bad and sounds good, you have measured the wrong thing, “ Daniel R. von Recklinghausen, Chief Engineer H.H. Scott, Inc. as cited on www.hhscott.com.

The quantitative tools for evaluating the accuracy or fidelity of an audio system have remained virtually unchanged since the commercial advent of high-fidelity analog systems in the late forties. At that time systems were very simple in concept and design, and fidelity criteria using stationary signals to measure static systems based upon frequency response and total harmonic or inter-modulation distortion were more than adequate. As audio systems became more sophisticated, dynamic analog signal processors such as Dolby® noise reduction, limiters, and compressors began to appear and be widely used. These dynamic systems began to stretch the limitations of the traditional static distortion measurement techniques.

The advent of digital-based systems added an additional layer of complexity to the audio recording chain. In order to maintain signal fidelity integrity, additional sources of error had to be considered and include A/D non-linearity, system clock noise, quantization noise, and anti-aliasing filter errors. System parameters can easily be dynamically controlled to achieve specific psychoacoustic effects. Examples of dynamic parameter control include adaptive frequency response contouring and noise management, dynamic compression and/or expansion, spatial manipulation and simulation (including introduction of multiple reverberation paths to recreate a desired acoustic environment), and editing techniques during the recording process to “enhance” an artist’s performance. As a result of the dynamic nature of the evolving audio technology, a more comprehensive measure of audio system fidelity is required.

Adaptive audio processing techniques were first applied to hearing aid devices. It soon became apparent that the traditional static measurement standards were inadequate. An example of an attempt to update a test standard originally written for static systems to address dynamic systems can be seen in the evolution of the ANSI standards for hearing aid devices, ANSI S3.22 [4]. The accepted measurement standard for hearing aid devices was established and presently maintained by the American National Standards Institute (ANSI) under the supervision of the Food and Drug Administration (FDA), since this is considered a medical device. When the standard is published it is a voluntary procedure until it is adopted by the FDA. From that point in time on, it is included in FDA regulations for testing hearing devices produced by manufacturers and to assist hearing dispensing professionals in fitting products to their patients. An attempt is made every 5 years to revise the current standard and in reality this process may take as long as 10 years. Because of this unusually long time delay the present standard may significantly lag current technology trends in the industry. This represents a significant problem that needs to be addressed.

The following is a brief history of development of the ANSI standards for hearing aids. ANSI established the first standards for hearing aid devices in 1976 (ANSI S3.22-1976). These devices used analog technology exclusively and were tested using stationary sinusoidal signals to measure frequency response, and total harmonic distortion. Subsequent ANSI standards published in 1987 (ANSI S3.22-1987), 1996 (ANSI S3.22-1996), and 2003 (ANSI S3.22-2003) continue to be based upon analog technology involving stationary sinusoidal signals to measure frequency response and total harmonic distortion. There were minor differences between these latter versions based upon attack times of the test signal, how the telecoil sensitivity was measured, and how the automatic gain control (AGC) features were set for the test procedure [4]. However, these minor differences have little significance with regard to the technique described in this investigation. None of these standards addresses analyzing dynamic systems in the presence of non-stationary test signals, but will probably be incorporated in later versions of ANSI S3.22.

Distortions generated from digital devices are not well understood and presently there is no consensus on a measurement standard. One research team [15] proposed a more general approach of measuring distortion which is not dependent on the internal processing techniques of the system under evaluation. This approach was an attempt to measure distortion produced by dynamic systems but requires additional theoretical and empirical development.

The proposed distortion error analysis technique described in this study develops a tool that models the random non-stationary nature of the actual audio signal and replaces traditional distortion and frequency response measurements and can be summarized as follows:

1. A Gaussian white noise sequence is chosen as the input test function with zero mean and unity variance.
2. Since this source is an uncorrelated process, i.e. the autocorrelation function is an impulse, it is desirable to provide some degree of correlation between time samples. This is required to allow for processing time delays that may occur in the audio system under test.
3. The coherence spectrum energy function between the input and the output signals of the audio system is computed. In order to minimize the dependency of time delays between the input and output signals, and to minimize aliasing errors, it is necessary to introduce enough correlation between time samples to allow for the processing delay time of the system under test. A good estimate for the bandwidth of the correlation filter is to set it to the bandwidth of the system under test.
4. The incoherence distortion energy spectrum function is defined as one minus the coherence energy spectrum function.
5. The average incoherence distortion is defined within the input signal passband.

Chapter 2

Introduction

Modeling of System Non-linearity

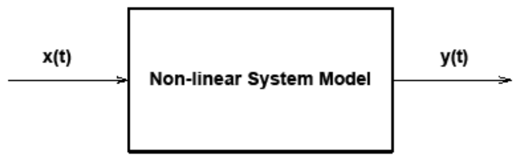

Fig: 1 Non-linear System Model

The non-linear system model will be represented in the time domain as an amplitude non-linearity in the form of a power series as

$$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + \ldots + a_n x^n(t) \tag{2.1}$$

Any system frequency dependency will be modeled as a transfer function following this amplitude non-linearity block. The coefficients a_0 through a_n are assumed to be time independent constants.

The dominant mode of non-linearity is dependent upon the electronic technology employed in the audio system under examination. In audio amplifiers, vacuum tube technology will have predominantly even-order power series non-linearities, solid state bi-polar technology will have predominantly odd-order non-linearities [2], [7], [12]. If this is a Class “AB” or Class “B” power amplifier, this will manifest itself predominantly as cross over distortion [11], [12]. Solid state uni-polar or FET technology will have even-order non-linearities, predominantly second order [11], [12].

In recording technology, analog magnetic tape will produce odd-order non-linearities, primarily third order [2], [7], [12]. This is a result of magnetic hysteresis in the recording medium. Digital recording introduces non-linearities in the A/D process including quantization errors. These are generally very difficult to characterize or model. In general, even-order harmonic distortion components are usually masked by speech or music since these sources inherently generate spectra that are rich in even harmonics. However, odd harmonic distortion components produced by odd-order non-linearities are much more easily detected by the human hearing process [2], [7], [12].

Introduction

Theory of Total Harmonic Distortion (THD) Measurements

Total Harmonic Distortion measurements date back to the early days of audio system analysis in the early 1900's. This measurement technique requires that a single deterministic stationary fixed amplitude and frequency sinusoid be applied to the system under test. If the system has any non-linearities, then new frequencies, i.e. harmonics, will be generated and will appear in the system output signal. The percent total harmonic distortion for a given test signal amplitude and frequency is defined as

$$\%THD = \left[\frac{\sum_{i=2}^{\infty} P_i}{\sum_{i=1}^{\infty} P_i} \right]^{1/2} \times 100 \qquad (2.2)$$

where P_i is the average power in the i^{th} harmonic for $i \geq 2$ and the average power in the fundamental for $i = 1$.

As both the amplitude and frequency of the test signal is varied, the measured %THD will vary. Usually, %THD is plotted as a function of frequency for a given amplitude level. Different levels are plotted on the same set of axes to form a family of THD curves. For most audio systems, the %THD will increase with both increasing test signal amplitude and frequency.

In a strict sense, %THD is not a valid measurement tool when multiple sinusoids of different amplitude and frequency are applied as the input test signal. For this situation, not only are harmonics of the input sinusoids generated, but also all sums and differences of all combinations of fundamental and harmonic frequencies are possible. Thus, harmonic distortion is not the only distortion component present in the output signal. A more appropriate description of distortion for this type of test signal would be to describe it as total distortion, which would be a combination of total harmonic and intermodulation distortion.

There are numerous problems associated with total harmonic distortion measurements:

1. THD measurements are based upon a single frequency steady-state deterministic test signal, i.e. a sinusoid. Actual speech or music is much more complex, representing many simultaneous sinsusoids, with amplitudes, frequencies, and phase components that are random in nature.
2. THD measurements are suitable only for static systems. Systems with dynamically changing frequency, phase, and amplitude parameters would not be accurately measured.
3. THD measurements generate an extremely inaccurate characterization of system non-linearity as the test frequency approaches the upper octave limit in a band limited system. In this region, all the higher harmonics are filtered by the system bandwidth limit and give a false, i.e. low, measure of THD. As a general guide line, the bandwidth of the system under test must be ten times greater than the test frequency for the THD measurement. Thus THD measurements near the upper bandwidth limit of the system under test become highly inaccurate.
4. It will be shown that as the complexity of the system non-linearity increases, it is possible that the THD measurement will decrease. By referring to Appendix A, it can be seen that even order non-linearities do not contribute a fundamental frequency component. However, odd order non-linearities do contribute to the fundamental frequency component. Thus if the non-linearity is rich in odd order terms, the fundamental

component will be larger than for any odd order. The result is that the denominator in the THD measure is growing faster than the numerator and the resultant THD measure reduces.

5. Since THD measurements are based on a sinusoid, the inherent limiting accuracy of any measurements is dependent on the spectral purity of the test sinusoid. Thus, the test sinusoidal oscillation must itself have a THD at least 1/10 that of the audio system that is under examination.

From a measurement standpoint, THD is relatively easy to obtain. The instrumentation required to measure THD is comparatively simple compared to other measures of distortion, and involves placing a notch filter tuned to the test frequency fundamental, allowing all harmonics to pass through and be applied to a power measurement circuit. The ratio of this harmonic power to the total power is the total harmonic distortion measurement.

Appendix A shows theoretical calculations for Total Harmonic Distortion for both single and dual sinusoidal input signals for first through fifth order system non-linearities.

Introduction

Theory of Incoherence Distortion Measurements

Incoherence distortion is a measure of the degree of dissimilarity in frequency, phase, and amplitude linearity between input and output signals in the system under test. This test can be performed for either a deterministic signal, such as a sinusoid, or a nondeterministic signal, such as the output of a stationary random process. Signals of the latter class allow for statistical measurement of dynamic systems, i.e. systems with bandwidths that are time varying with input signal dynamics, as well as traditional spectrum performance measurements.

The coherence power spectral density is defined as $\eta_{XY}(f)$,

$$\eta_{XY}(f) = \frac{S_{XY}(f)}{S_X(f)S_Y(f)} = |\eta_{XY}(f)|e^{j\phi_{XY}(f)}$$

where $\phi_{XY}(f)$ is the phase of the coherence. The incoherence distortion power spectral density is defined as $\gamma_{XY}(f)$, where

$$0 \le |\gamma_{XY}(f)|^2 = 1 - |\eta_{XY}(f)|^2 = 1 - \frac{|S_{XY}(f)|^2}{[S_X(f)S_Y(f)]} \le 1 \qquad (2.3)$$

where $S_{XY}(f)$ is the cross power spectral density between the system input, $x(t)$, and the system output, $y(t)$. $S_X(f)$ is the power spectral density of the system input signal and $S_Y(f)$ is the power spectral density of the system output signal.

The phase of $\eta_{XY}(f)$, $\phi_{XY}(f)$, is a measure of the phase coherence or linearity of the system under test.

The average incoherence distortion over a band limit is defined as

$$0 \le \bar{\gamma}_{XY} = \left(\frac{1}{f_H - f_L} \right) \int_{f_L}^{f_H} \left| \gamma_{XY}(f) \right|^2 df \le 1 \tag{2.4}$$

where f_L is the lower frequency limit and f_H is the upper frequency limit of the audio system under examination, and can be written as a percent.

The incoherence distortion power spectrum shows the spectral distribution or density of the distortion components introduced by the system under examination, and clearly shows the frequency region in the system bandwidth where there is least distortion. The incoherence distortion measurement is sensitive to not only non-linearities introduced by the system under test but also frequency and phase response deviations. Thus a single measurement parameter can replace numerous previously accepted performance measurement parameters.

The signal-to-distortion ratio as a function of frequency is defined as,

$$\left(S/D \right)_{dB}(f) = 10 \log_{10} \frac{1 - \left| \eta_{XY}(f) \right|^2}{\left| \eta_{XY}(f) \right|^2} dB \tag{2.5}$$

Chapter 3

THD Measurement Program

Description of THD Analysis Program as Implemented in MATLAB®

An m-file function in MATLAB® is a program routine that accepts input arguments and returns output arguments. Once the m-file function is invoked by typing the name of the file at the MATLAB® command prompt, the program routine is loaded into memory. The first task of the program, “compthdpercent21”, is to ask the user several questions concerning the system to be tested and the input signals applied to the system under test. First, the program asks for the initial value of the number of sample points (NPTS) by setting z.

$$NPTS = 2^{z} \tag{3.1}$$

NPTS might be changed later on by the program in order to select an optimal frequency resolution. This is accomplished by ensuring that the program has at least 200 frequency samples between DC and the lowest sinusoidal input frequency. The upper limit of NPTS is set by the resources of the computer system. Second, the program asks for the sampling frequency (Hz), Fs. Third the program asks the user for the number of sinusoidal vectors. The program internally establishes the minimum and maximum boundaries for the frequency of the sinusoidal vectors to be entered next. Fourth, the program asks for the frequency and peak amplitude for each sinusoidal vector within the minimum and maximum bounds established by the program. From the lowest frequency entered the program internally calculates the lower and upper bounds of NPTS and makes a determination if NPTS needs to be adjusted from the initial value entered. If the program readjusts NPTS, as described previously, then the new NPTS value will be stored and displayed on the screen. Fifth, the program asks for the coefficients of the polynomial representation of the non-linearity from DC to fifth order. From this information, the program creates an array or matrix of the sinusoidal vectors and sums these to form a composite 1-D input signal if it doesn't already exist.

The program computes the power spectral density (PSD) of the input signal by taking the Discrete Fourier Transform on the composite 1-D input signal and dividing by NPTS. The absolute value is taken of the result and then raised to the second power. A sample result for a single fundamental frequency at 700 Hz is plotted in MATLAB® and titled "Input Power Spectral Density vs. Frequency," shown in figure 2.

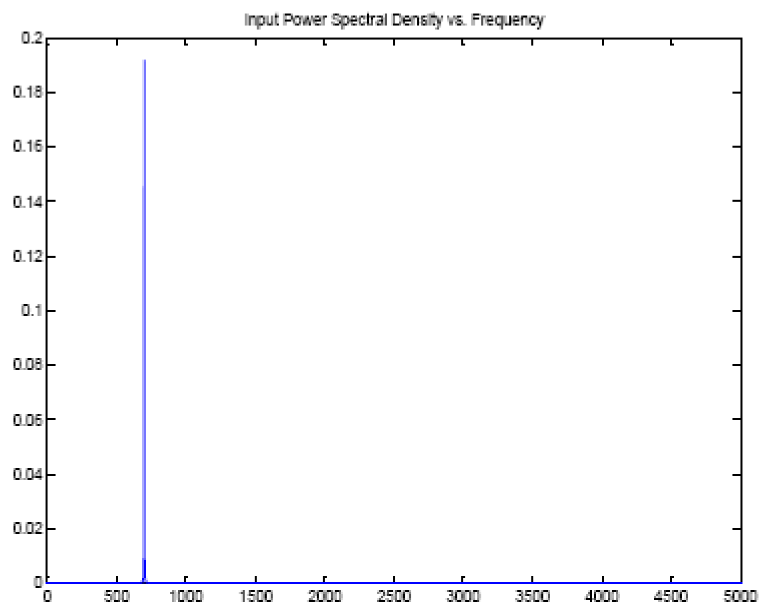

Fig. 2 Input Power Spectral Density vs. Frequency for 700 Hz Sinusoid

The program scans for a frequency with maximum amplitude in the input PSD. This is done by finding the MATLAB® index equivalent frequency corresponding to the maximum amplitude and storing this value. Then this maximum amplitude is zeroed out so that the program can scan for the next largest amplitude and corresponding MATLAB® index equivalent frequency. This process continues until the program reaches the total number of applied input sinusoidal vectors. The fundamental frequencies are identified, stored, and displayed in Hz.

The program contains an adaptive process to compute the mean and variance on a shrinking window across the input frequency spectrum to establish the input noise floor. The mechanics of the adaptive process breaks up the approach two different ways. First, if there is a single fundamental frequency component present in the input PSD, the program breaks up the input PSD into two windows, shown in figure 3. Window 1 is a symmetrically shrinking window from 1 to the $f_{fundamental}$ + 1 MATLAB® index equivalent frequency. Window 2 is an asymmetrical shrinking window moving from the $f_{fundamental}$ + 1 to (NPTS/2) + 1 MATLAB® index equivalent frequency. As windows 1 and 2 are shrinking, the program computes and stores the mean and variance every time the window shrinks in a matrix for further processing. The program computes the derivative of the variance and then the magnitude of the derivative of the variance for windows 1 and 2. The program finds the maximum value for the magnitude of the derivative of the variance for windows 1 and 2. Then the program finds the MATLAB® index equivalent frequency that is less than or equal to 1% of the maximum value for the magnitude of the derivative of the variance for windows 1 and 2. This is the threshold MATLAB® index equivalent frequency which is used to look up and store the mean value associated at this threshold MATLAB® index equivalent frequency. The maximum mean value between windows 1 and 2 is then used to set the input noise floor.

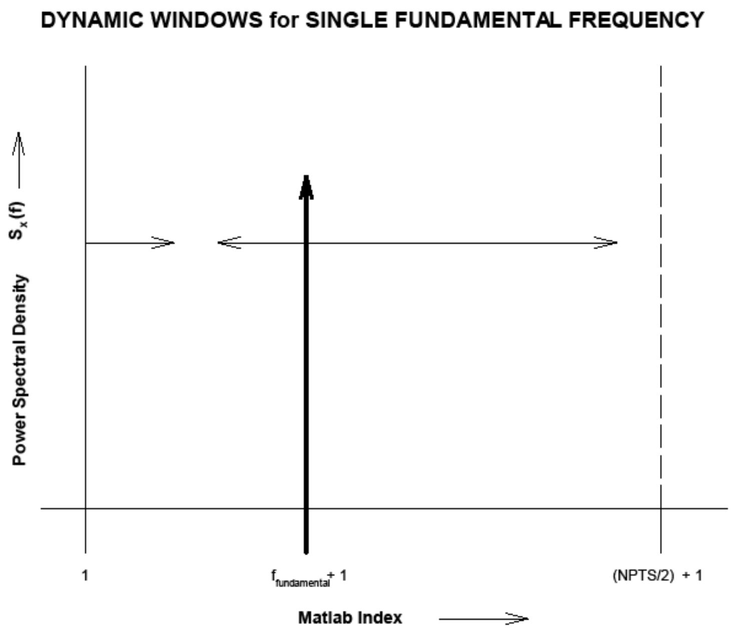

Fig. 3 Dynamic Windows for Single Fundamental Frequency

If there are n number fundamental frequency components present in the input PSD, the program breaks up the input PSD into n + 1 number of windows. There exists n number of symmetrically shrinking windows and one asymmetrical shrinking window. Figure 4 illustrates a case where there are three fundamental frequency components in the input PSD. Therefore, there will be a total of four windows. Window 1 is a symmetrically shrinking window from 1 to the $f_{fundamental\ 1}$ +1 MATLAB® index equivalent frequency. Window 2 is a symmetrically shrinking window from the $f_{fundamental\ 1}$ +1 to the $f_{fundamental\ 2}$ +1 MATLAB® index equivalent frequency. Window 3 is a symmetrically shrinking window from $f_{fundamental\ 2}$ +1 to the $f_{fundamental\ 3}$ +1 MATLAB® index equivalent frequency. Window 4 is an asymmetrical shrinking window moving from the $f_{fundamental\ 3}$ to (NPTS/2) + 1 MATLAB® index equivalent frequency. As all the windows are shrinking, the program computes and stores the mean and variance every time the windows shrink in a matrix for further processing. The program computes the derivative of the variance and then the magnitude of the derivative of the variance for all the windows. The program finds the maximum value for the magnitude of the derivative of the variance for all the windows. The program then finds the MATLAB® index equivalent frequency that is less than or equal to 1% of the maximum value for the magnitude of the derivative of the variance for all the windows. This is the threshold MATLAB® index equivalent frequency which is used to look up and store the mean value associated at this threshold MATLAB® index equivalent frequency. The maximum mean value between all the windows is then used to set the input noise floor.

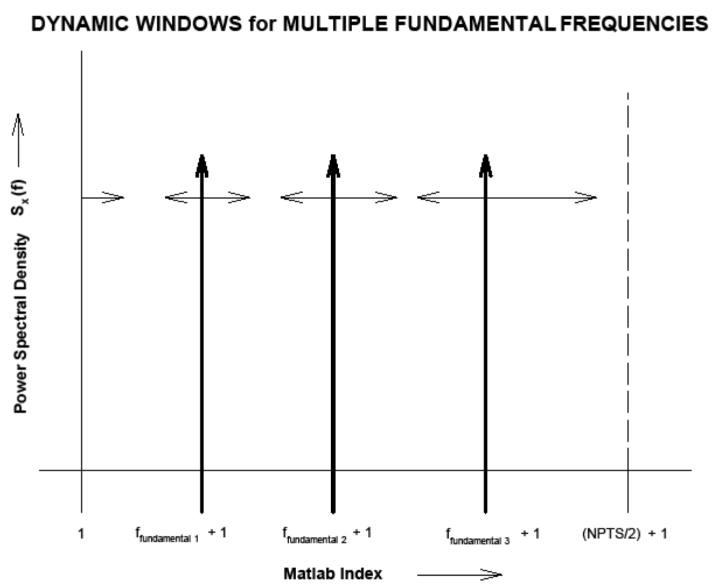

Fig. 4 Dynamic Windows for Multiple Fundamental Frequencies

The program then generates and plots the system non-linear model transfer function. First and third order amplitude transfer functions are shown in Fig. 5 and Fig. 6.

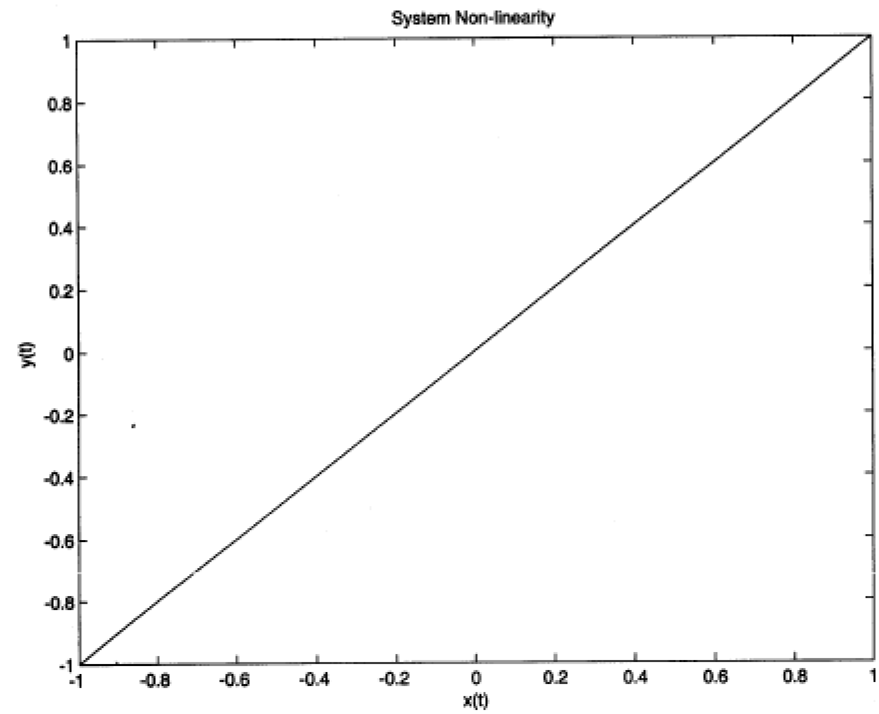

Fig. 5 Linear System Gain

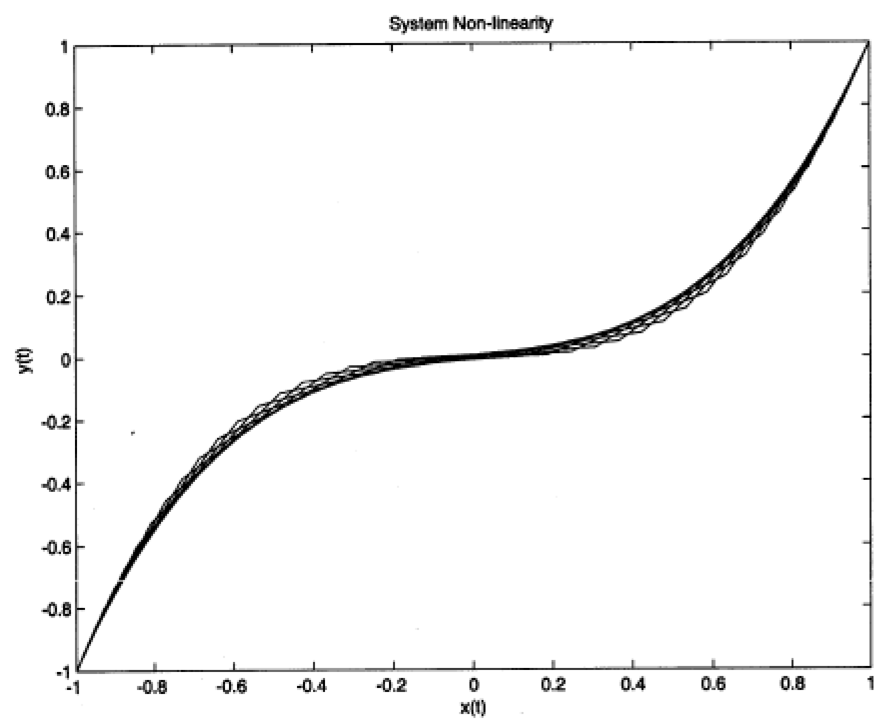

Fig. 6 Third Order Non-linear System Gain

The program also generates and plots the output PSD vs. frequency for a first and third order non-linearity, shown in Fig. 7 and Fig. 8.

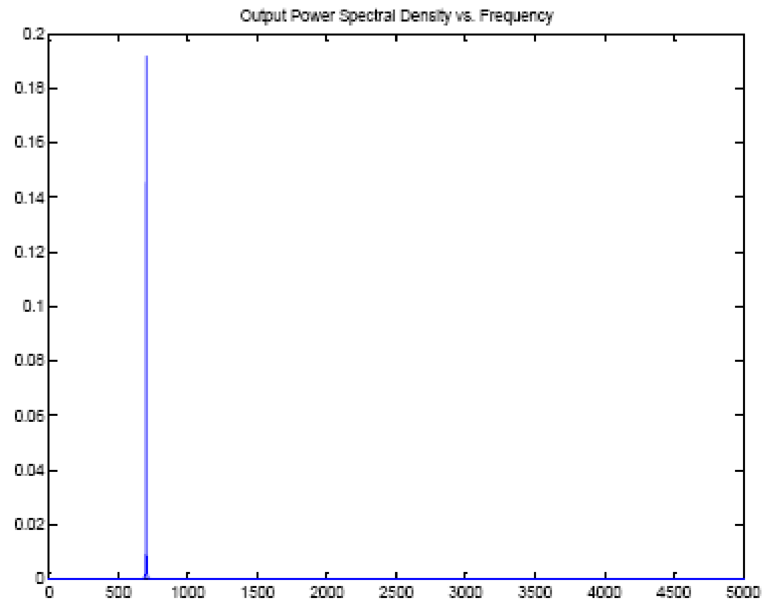

Fig. 7 Output Power Spectral Density For Linear System

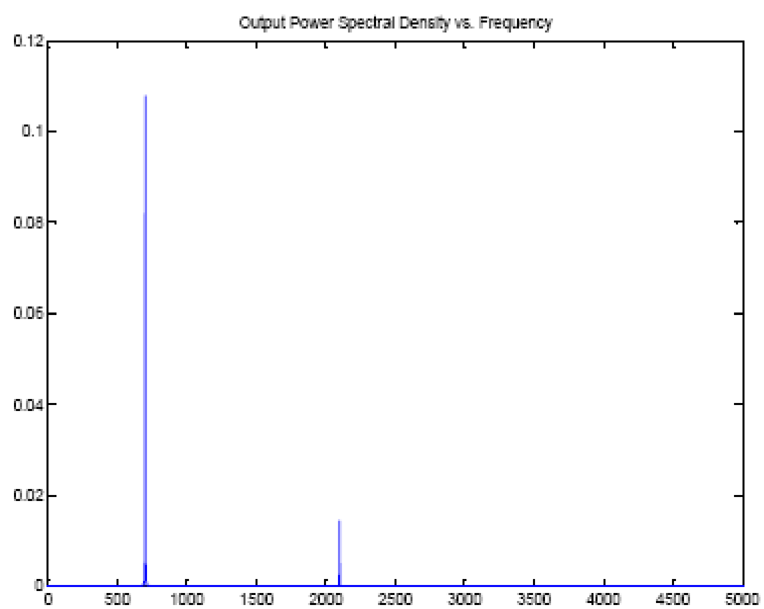

Fig. 8 Output Power Spectral Density For Third Order Non-linear System

The program sets the input noise floor equal to the output noise floor and begins to find frequencies whose amplitude are greater than the output noise floor. Some frequency components will form a distribution while others will be a single impulse. The program will reduce a spectral distribution to an impulse centered at the centroid of the distribution. It does this by looking for spectral distributions right next to each other and forming a group around that cluster ignoring DC. Each cluster becomes a new row in a matrix allowing the program to count the number of frequency components and to reduce spectral distributions to an impulse centered at the centroid of the distribution. While the amplitudes are summed to form a composite value, the spectral distributions are reduced by equation C6 of Appendix C. The normalized output frequencies are then displayed on the screen by the program. Once the output frequencies are known the program then creates a matrix of all possible combinations by dividing the normalized output frequencies by the fundamental frequencies in the input PSD. The program then compares the results in this matrix to known harmonics from the second to the tenth and also compares the result to the input fundamental frequency or frequencies. If two or more input frequencies are multiples of each other, their corresponding harmonics can not be mapped and the program ends. The program would have to be restarted and new frequencies selected to continue. As long as the selected input frequencies are not multiples of each other, the program then begins to compute the harmonic power by summing all of the powers in each harmonic. Each individual harmonic power can be found by squaring the consolidated amplitude for each harmonic. The fundamental power is calculated by summing the squares of each fundamental's amplitude. The total power is found by summing the fundamental power and the harmonic power. As shown in equation 2.2, the % Total Harmonic Distortion is found by taking the square root of the harmonic power divided by the total power and taking that result and multiplying by 100. The harmonic power, fundamental power, total power, and % Total Harmonic Distortion are all displayed on the screen.

THD Measurement Program

Results of Test Signals and System Non-linearity

Experimental Conditions:
waveshape = sinusoid, amplitude = 1, frequency = 700 Hz, sample frequency = 10 kHz, NPTS = 8192
(Refer to Equation 2.1)

Coefficients of nonlinearity a_0	a_1	a_2	a_3	a_4	a_5	**% Total Harmonic Distortion**
0	1	0	0	0	0	0.00
0	0	1	0	0	0	100.00
0	0	0	1	0	0	31.52
0	0	0	0	1	0	100.00
0	0	0	0	0	1	45.34
0	1	1	0	0	0	44.77
0	1	1	1	0	0	30.43
0	1	1	1	0	1	30.30
0	1	1	1	1	0	51.05
0	1	1	1	1	1	43.78
0	1	1	0	1	0	71.03
0	1	1	0	0	1	34.27
0	1	1	0	1	1	54.55
0	1	0	1	1	1	30.67
0	1	0	0	0	1	19.17
0	1	0	0	1	1	34.92
0	1	0	1	1	0	31.10
0	1	0	1	0	1	23.14
0	1	0	1	0	0	14.08
0	0	1	1	1	1	64.37
0	0	1	0	0	1	68.86
0	0	1	0	1	0	100.00
0	0	1	1	0	0	59.79
0	0	1	1	0	1	48.14
0	0	1	1	1	0	81.11
0	0	1	0	1	1	86.12
0	0	0	1	1	1	48.63
0	0	0	1	0	1	38.01
0	0	0	1	1	0	60.71
0	0	0	0	1	1	69.63

Table 1: Tabulated % THD Measurements for All Combinations of First thru Fifth Order Non-linearities

Chapter 4

Incoherence Measurement Program

Description of Incoherence Program as Implemented in Simulink®

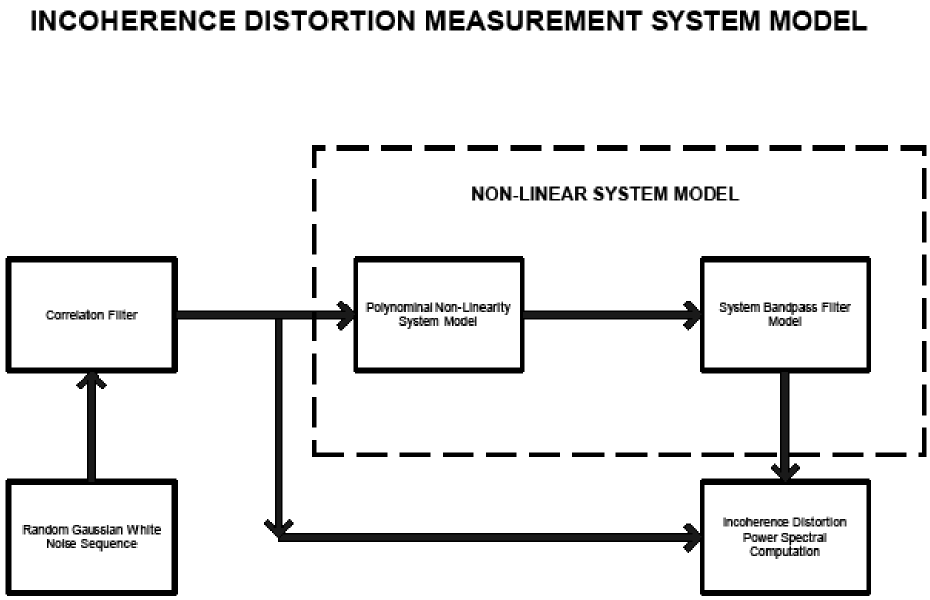

Fig. 9 Incoherence Distortion Measurement Model

The Simulink® model for the incoherence measurement technique is shown in Figure 9. The signal source is a Gaussian white noise sequence of zero mean and unity variance. This output is applied to the input of a low pass filter. The function of this filter is to correlate the output sequence of the Gaussian noise source and to minimize aliasing errors. Figure 10 displays the autocorrelation function of the random sequence at the output of the correlation filter. The output of the low pass filter is applied to the input of the model representing the system under evaluation. This input signal is also applied to one input of the coherence power spectrum estimation block, and to the input of a block representing the non-linear model for the system under test. The output of the non-linear model is applied to the input of a filter model representing the frequency and phase response model for the system under test. Figure 11 shows the frequency and phase response of this filter. The output of this block is applied to the other input of the coherence power spectrum estimation block. The output of the coherence block is then subtracted from one to form the incoherence distortion power spectrum. Figure 12 shows the incoherence distortion power spectral density for a linear system with a 100 Hz Bandpass filter. Figures 13 through 16 show the incoherence distortion power spectral density for second through fifth order non-linearities respectively with a 100 Hz Bandpass filter.

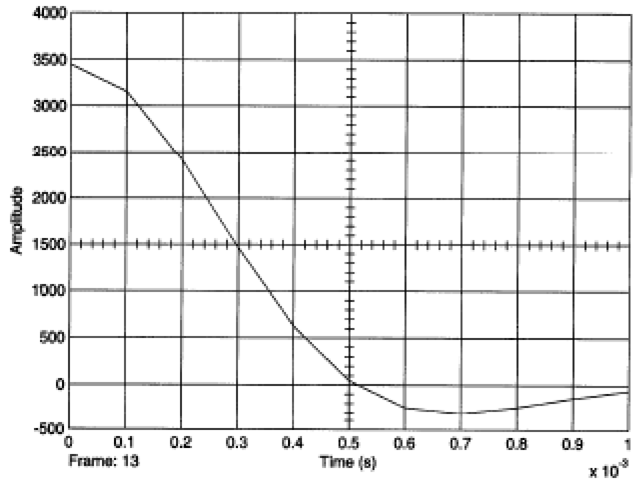

Fig. 10 Autocorrelation of Signal at Output of Correlation Filter

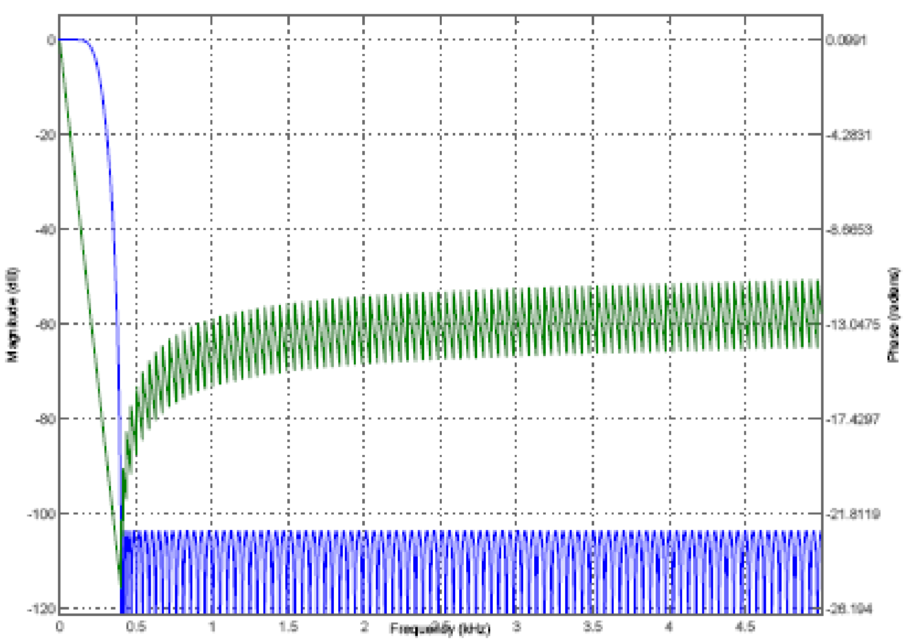

Fig. 11 System Bandpass Filter Model

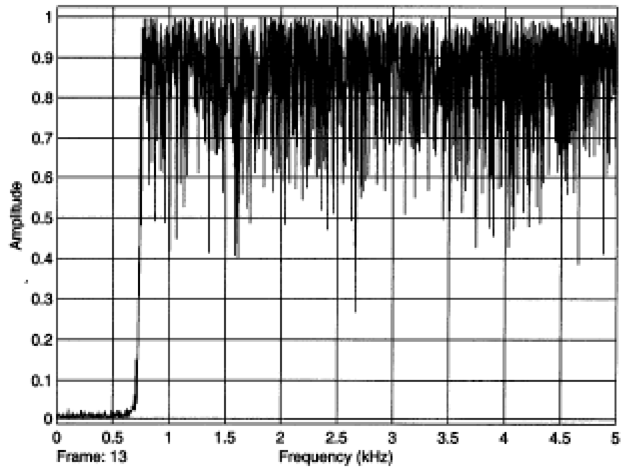

Fig. 12 Incoherence Distortion Power Spectral Density
for a Linear System with a 100 Hz Bandpass Filter

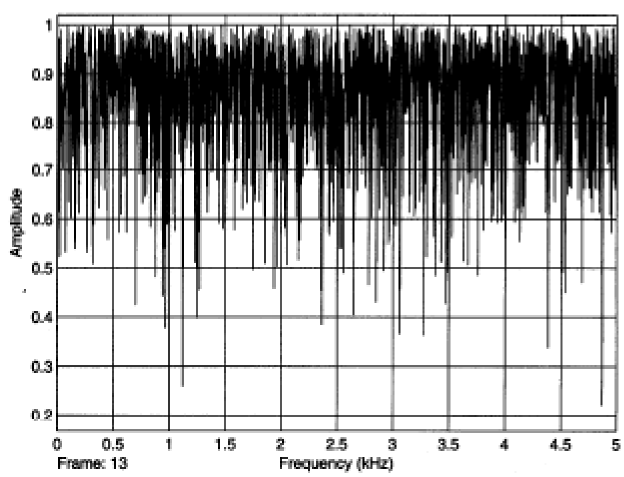

Fig. 13 Incoherence Distortion Power Spectral Density for a
Second Order Non-linear System with a 100 Hz Bandpass Filter

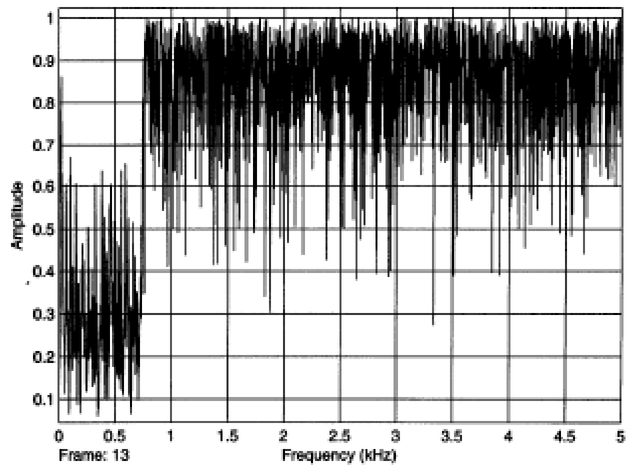

Fig. 14 Incoherence Distortion Power Spectral Density for a Third Order Non-linear System with a 100 Hz Bandpass Filter

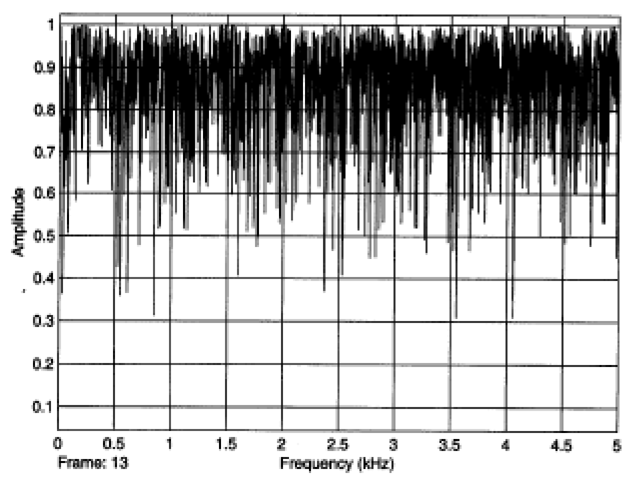

Fig. 15 Incoherence Distortion Power Spectral Density for a Fourth Order Non-linear System with a 100 Hz Bandpass Filter

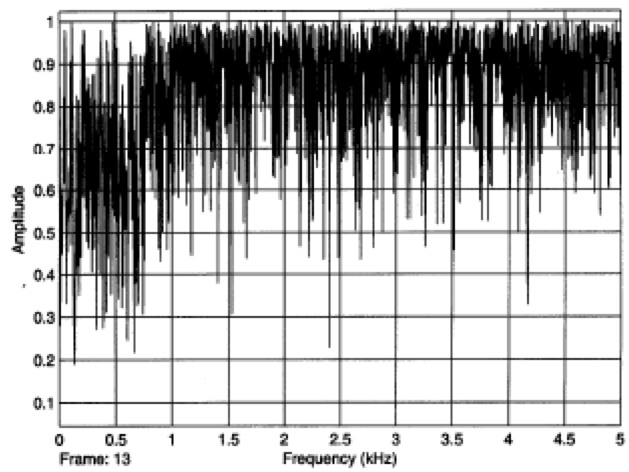

Fig. 16 Incoherence Distortion Power Spectral Density for a Fifth Order Non-linear System with a 100 Hz Bandpass Filter

Experimental Conditions:
waveshape = Gaussian Random Source, mean = 0, variance = 1, Fs = 10000 Hz, samples/frame = 16384

(Refer to Equation 2.1)

Coefficients of nonlinearity						**% Average Incoherence Distortion**
a_0	a_1	a_2	a_3	a_4	a_5	**within test signal pass band**
0	1	0	0	0	0	0.4613
0	0	1	0	0	0	72.72
0	0	0	1	0	0	29.26
0	0	0	0	1	0	71.23
0	0	0	0	0	1	50.07
0	1	1	0	0	0	23.69
0	1	1	1	0	0	19.40
0	1	1	1	0	1	21.42
0	1	1	1	1	0	32.15
0	1	1	1	1	1	35.10
0	1	1	0	1	0	48.98
0	1	1	0	0	1	28.94
0	1	1	0	1	1	39.70
0	1	0	1	1	1	24.19
0	1	0	0	0	1	18.78
0	1	0	0	1	1	28.17
0	1	0	1	1	0	20.41
0	1	0	1	0	1	21.86
0	1	0	1	0	0	5.71
0	0	1	1	1	1	59.36
0	0	1	0	0	1	51.17
0	0	1	0	1	0	74.13
0	0	1	1	0	0	62.69
0	0	1	1	0	1	51.06
0	0	1	1	1	0	68.30
0	0	1	0	1	1	66.30
0	0	0	1	1	1	51.21
0	0	0	1	0	1	37.22
0	0	0	1	1	0	51.54
0	0	0	0	1	1	57.23

Table 2: Tabulated % Incoherence Distortion for All Combinations of First thru Fifth Order Non-linearities

Chapter 5
Discussion of Results and Conclusions

I. Total Harmonic Distortion

Table 1 shows the results of an analysis of the total harmonic distortion for the non-linear system described in Equation 2.1. All relevant combinations of the coefficients were examined, assuming values of either zero or one, in order to better see the interaction of powers of x in Equation 2.1 on system non-linearitites. This table verifies the calculated results of Appendix A. As noted earlier, as various order non-linearities are added together, the % THD measurement may actually drop. The theoretical values matched the actual calculated values within expected computational errors.

II. Incoherence

Table 2 which tabulates the results of % incoherence distortion within the passband of the test signal shows that for single order non-linearities the even order non-linearities produce much higher distortion than odd-order non-linearitities. This is to be expected since even-order non-linearities are producing only positive polarity signals. This results in extreme damage to the original signal. Odd-order non-linearities produce significantly less distortion than even-order non-linearities but the distortion monotonically increases with increasing odd-order.

The incoherence measure is much more sensitive to any type of amplitude, phase, or frequency non-linearity than traditional % THD measurements. This makes it a very effective tool for characterizing audio system fidelity and replacing traditional measures of frequency and phase response and THD and IM measurements with a single measurement tool.

Chapter 6
Areas for Future Investigation

I. Multichannel Systems

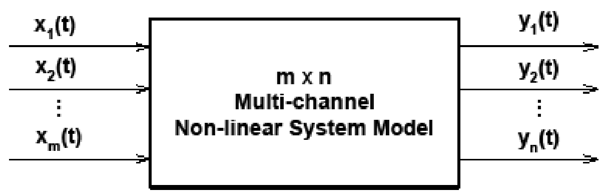

Fig. 17 Multi-channel Non-linear System Model

Figure 17 shows the block diagram for a multi-channel audio system model. The dimension of the input signal $x(t)$ is m, while that of the output signal $y(t)$ is n In other words, there are m input signal channels and n output signal channels.

Distortion modeling of multi-band systems could be represented as

$$\boldsymbol{\gamma}(\mathbf{f}) = \left[\begin{array}{cccc|cc} \gamma_{X_1Y_1}(f) & \gamma_{X_1Y_2}(f) & & \gamma_{X_1Y_m}(f) & \cdots & \gamma_{X_1Y_n}(f) \\ \gamma_{X_2Y_1}(f) & \gamma_{X_2Y_2}(f) & & \vdots & & \\ \vdots & \vdots & \ddots & \vdots & \ddots & \\ \gamma_{X_mY_1}(f) & \gamma_{X_mY_2}(f) & & \gamma_{X_mY_m}(f) & & \gamma_{X_mY_n}(f) \end{array}\right] \tag{6.1}$$

Where the major diagonal components of the partitioned matrix represent the incoherence distortion power spectrum for each primary channel and the off diagonal components represent the interchannel cross coupling (i.e. interchannel cross talk or channel separation) incoherence distortion power spectrum. The off diagonal components are a measure of the interchannel cross coupling distortion.

This represents an area of intense activity involving multichannel surround formats. This technique represents a very effective tool to model the interchannel performance of a multichannel system and collect system performance data very rapidly. This procedure could be performed periodically as part of a system maintenance schedule for proof of performance.

II. Frequency Weighting

The incoherence distortion measure could be enhanced by adding frequency contouring to the incoherence distortion power spectral density to allow for the dynamically changing frequency response of the human ear as a function of SPL. In effect, the Fletcher-Munsen curves could be used to continuously shape the incoherence distortion power spectrum as a function of SPL. This would give a more accurate measure of perceived distortion.

III. Dynamically Changing Input Signal Amplitudes

Further work needs to be performed in the application of a Gaussian white noise sequence with a slowly ramped variance. This would allow measurement of system performance parameters over a contour of input signal power levels rather than at fixed input power levels.

IV. Phase Incoherence Distortion

Much more work is required to explore the benefits of measuring the phase angle spectrum of the incoherence distortion function and relating it to system non-linearities.

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Appendix A

Theoretical Calculation of Percent Total Harmonic Distortion (%THD)

Percent total harmonic distortion is defined as,

$$\%THD = \left[\frac{\sum_{i=2}^{\infty} P_i}{\sum_{i=1}^{\infty} P_i} \right]^{1/2} \times 100 \qquad \text{(A1)}$$

where P_i is the average power in the i^{th} harmonic for $i \geq 2$ and the average power in the fundamental for $i = 1$.

Note: The average power associated with the DC component, (i.e. i = 0, P_0), will be ignored since this does not contribute to any audible information.

I. Initially, a single sinusoidal frequency excitation will be assumed,

where

$$x = \sin A \qquad \text{(A2)}$$

For a 1st Order Linear System

$$y = x = \sin A \qquad \text{(A3)}$$

since no harmonic power is present,

$$\%THD = \left[\frac{0}{0.5}\right]^{1/2} \times 100 = 0\% \tag{A4}$$

For a 2nd Order Non-linear System

$$y = x^2 = \sin^2 A = \frac{1}{2}(1 - \cos 2A)$$

$$= \frac{1}{2} - \frac{1}{2}\cos 2A \tag{A5}$$

thus, from Eq. A1 and Eq. A5

$$\%THD = \left[\frac{\left(\frac{1}{2\sqrt{2}}\right)^2}{\left(\frac{1}{2\sqrt{2}}\right)^2}\right]^{1/2} \times 100 = 100\% \tag{A6}$$

For a 3rd Order Non-linear System

$$y = x^3 = \sin^3 A = \sin A\left[\frac{1}{2}(1 - \cos 2A)\right]$$

$$= \sin A\left[\frac{1}{2}(1 - \cos 2A)\right]$$

$$= \frac{1}{2}\sin A - \frac{1}{2}\sin A \cos 2A$$

$$= \frac{1}{2}\sin A - \frac{1}{2}\left[\frac{1}{2}\sin 3A + \frac{1}{2}\sin(-A)\right]$$

$$= \frac{1}{2}\sin A - \frac{1}{4}\sin 3A - \frac{1}{4}\sin(-A)$$

$$= \frac{1}{2}\sin A - \frac{1}{4}\sin 3A + \frac{1}{4}\sin A$$

$$= \frac{3}{4}\sin A - \frac{1}{4}\sin 3A \tag{A7}$$

thus, from Eq. A1 and Eq. A7

$$\%THD = \left[\frac{\left(\frac{1}{4\sqrt{2}}\right)^2}{\left(\frac{3}{4\sqrt{2}}\right)^2 + \left(\frac{1}{4\sqrt{2}}\right)^2}\right]^{1/2} = \left(\frac{1}{10}\right)^{1/2} \times 100 \cong 31.62\% \tag{A8}$$

For a 4th Order Non-linear System

$$y = x^4 = \sin^4 A = \sin^2 A \sin^2 A$$

$$= \frac{1}{2}(1 - \cos 2A)\frac{1}{2}(1 - \cos 2A)$$

$$= \frac{1}{4}\left(1 - 2\cos 2A + \cos^2 2A\right)$$

$$= \frac{1}{4} - \frac{1}{2}\cos 2A + \frac{1}{4}\cos^2 2A$$

$$= \frac{1}{4} - \frac{1}{2}\cos 2A + \frac{1}{4}\left[\frac{1}{2}(1 + \cos 4A)\right]$$

$$= \frac{1}{4} - \frac{1}{2}\cos 2A + \frac{1}{8} + \frac{1}{8}\cos 4A$$

$$= \frac{3}{8} - \frac{1}{2}\cos 2A + \frac{1}{8}\cos 4A \tag{A9}$$

thus, from Eq. A1 and Eq. A9

$$\%THD = \left[\frac{\left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{8\sqrt{2}}\right)^2}{\left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{8\sqrt{2}}\right)^2} \right]^{1/2} = 100\% \tag{A10}$$

For a 5th Order Non-linear System

$$y = x^5 = \sin^5 A = \sin A \sin^4 A$$

$$= \sin A \left[\frac{3}{8} - \frac{1}{2}\cos 2A + \frac{1}{8}\cos 4A \right]$$

$$= \frac{3}{8}\sin A - \frac{1}{2}\sin A \cos 2A + \frac{1}{8}\sin A \cos 4A$$

$$= \frac{3}{8}\sin A - \frac{1}{2}\left[\frac{1}{2}\sin 3A + \frac{1}{2}\sin(-A) \right] + \frac{1}{8}\left[\frac{1}{2}\sin 5A + \frac{1}{2}\sin(-3A) \right]$$

$$= \frac{3}{8}\sin A - \frac{1}{4}\sin 3A + \frac{1}{4}\sin A + \frac{1}{16}\sin 5A - \frac{1}{16}\sin 3A$$

$$= \frac{5}{8}\sin A - \frac{5}{16}\sin 3A + \frac{1}{16}\sin 5A \tag{A11}$$

thus, from Eq. A1 and Eq. A11

$$\%THD = \left[\frac{\left(\frac{5}{16\sqrt{2}}\right)^2 + \left(\frac{1}{16\sqrt{2}}\right)^2}{\left(\frac{5}{8\sqrt{2}}\right)^2 + \left(\frac{5}{16\sqrt{2}}\right)^2 + \left(\frac{1}{16\sqrt{2}}\right)^2} \right]^{1/2} \times 100$$

$$= \left[\frac{\frac{25}{512} + \frac{1}{512}}{\frac{100}{512} + \frac{25}{512} + \frac{1}{512}} \right]^{1/2} \times 100$$

$$= \left(\frac{\frac{26}{512}}{\frac{126}{512}} \right)^{1/2} \times 100$$

$$= \left(\frac{26}{126} \right)^{1/2} \times 100$$

$$= \left(\frac{13}{63} \right)^{1/2} \times 100$$

$$\cong 45.43\% \qquad \text{(A12)}$$

II. As a second case, two sinusoidal frequency excitations will be assumed,

$$x = \sin A + \sin B \qquad \text{(A13)}$$

For a 1st Order Linear System

$$y = x = \sin A + \sin B \qquad \text{(A14)}$$

Thus from Eq. A1 and Eq. A14,

$$\%THD = \left[\frac{0}{1} \right]^{1/2} \times 100 = 0\% \qquad \text{(A15)}$$

For a 2nd Order Non-linear System

$$y = x^2 = (\sin A + \sin B)^2$$

$$y = (x+z)^2 = (\sin A + \sin B)^2 = \sin^2 A + \sin^2 B + 2\sin A \sin B$$

$$= \frac{1}{2}(1-\cos 2A) + \frac{1}{2}(1-\cos 2B) + 2\sin A \sin B$$

$$= \frac{1}{2}(1-\cos 2A) + \frac{1}{2}(1-\cos 2B) + \cos(A-B) - \cos(A+B)$$

$$= \frac{1}{2} - \frac{1}{2}\cos 2A + \frac{1}{2} - \frac{1}{2}\cos 2B + \cos(A-B) - \cos(A+B)$$

$$= 1 - \frac{1}{2}\cos 2A - \frac{1}{2}\cos 2B + \cos(A-B) - \cos(A+B) \quad \text{(A16)}$$

Thus from Eq. A1 and Eq. A16,

$$\%THD = \left[\frac{\left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2}{\left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2}\right]^{1/2} \times 100 = 100\% \quad \text{(A17)}$$

For a 3rd Order Non-linear System

$$y = x^3 = (\sin A + \sin B)^3$$

$$= (\sin A + \sin B)(\sin A + \sin B)^2$$

$$= (\sin A + \sin B)(\sin^2 A + \sin^2 B + 2\sin A \sin B)$$

$$= \sin^3 A + \sin A \sin^2 B + 2\sin^2 A \sin B + \sin^2 A \sin B + \sin^3 B + 2\sin A \sin^2 B \quad \text{(A18)}$$

$$= \sin^3 A + 3\sin A \sin^2 B + 3\sin^2 A \sin B + \sin^3 B$$

Where the first term in Eq. A8 is

$$\sin^3 A = \sin A \sin^2 A = \sin A\left[\frac{1}{2}(1-\cos 2A)\right]$$

$$= \frac{1}{2}\sin A - \frac{1}{2}\sin A \cos 2A$$

$$= \frac{1}{2}\sin A - \frac{1}{2}\left[\frac{1}{2}\sin 3A + \frac{1}{2}\sin(-A)\right]$$

$$= \frac{1}{2}\sin A - \frac{1}{4}\sin 3A + \frac{1}{4}\sin A$$

$$= \frac{3}{4}\sin A - \frac{1}{4}\sin 3A \qquad \text{(A19)}$$

Where the fourth term in Eq. A18 is

$$\sin^3 B = \frac{3}{4}\sin B - \frac{1}{4}\sin 3B \qquad \text{(A20)}$$

Where the second term in Eq. A18 is

$$3\sin A \sin^2 B = 3\sin A\left[\frac{1}{2}(1-\cos 2B)\right]$$

$$= \frac{3}{2}\sin A - \frac{3}{2}\sin A \cos 2B$$

$$= \frac{3}{2}\sin A - \frac{3}{4}[\sin(A+2B) + \sin(A-2B)]$$

$$= \frac{3}{2}\sin A - \frac{3}{4}\sin(A+2B) - \frac{3}{4}\sin(A-2B) \qquad \text{(A21)}$$

Where, the third term in Eq. A18 is

$$3\sin^2 A \sin B = \frac{3}{2}\sin B - \frac{3}{4}\sin(2A+B) - \frac{3}{4}\sin(-2A+B) \qquad \text{(A22)}$$

Thus by combining Eq. A19 thru Eq. A22 from Eq. A18

$$y = x^3 = \frac{3}{4}\sin A - \frac{1}{4}\sin 3A + \frac{3}{4}\sin B - \frac{1}{4}\sin 3B + \frac{3}{2}\sin A - \frac{3}{4}\sin(A+2B)$$

$$-\frac{3}{4}\sin(A-2B) + \frac{3}{2}\sin B - \frac{3}{4}\sin(2A+B) - \frac{3}{4}\sin(-2A+B)$$

$$y = x^3 = \frac{9}{4}\sin A + \frac{9}{4}\sin B - \frac{1}{4}\sin 3A - \frac{1}{4}\sin 3B - \frac{3}{4}\sin(A+2B)$$

$$-\frac{3}{4}\sin(A-2B) - \frac{3}{4}\sin(2A+B) - \frac{3}{4}\sin(-2A+B) \tag{A23}$$

Thus from Eq. A1 and Eq. A23,

$$\%THD = \left[\frac{\left(\frac{1}{4\sqrt{2}}\right)^2 + \left(\frac{1}{4\sqrt{2}}\right)^2}{\left(\frac{9}{4\sqrt{2}}\right)^2 + \left(\frac{9}{4\sqrt{2}}\right)^2 + \left(\frac{1}{4\sqrt{2}}\right)^2 + \left(\frac{1}{4\sqrt{2}}\right)^2}\right]^{1/2} \times 100$$

$$= \left[\frac{2\left(\frac{1}{4\sqrt{2}}\right)^2}{2\left(\frac{9}{4\sqrt{2}}\right)^2 + 2\left(\frac{1}{4\sqrt{2}}\right)^2}\right]^{1/2} \times 100$$

$$= \left[\frac{\frac{1}{16}}{\frac{81}{16} + \frac{1}{16}}\right]^{1/2} \times 100$$

$$= \left[\frac{1}{82}\right]^{1/2} \times 100$$

$$\cong 11.04\% \tag{A24}$$

For a 4th Order Non-linear System (with 2 sinusoids)

$$y = x^4 = (\sin A \sin B)^4$$

$$= (x^2)^2$$

$$= \left[1 - \frac{1}{2}\cos 2A - \frac{1}{2}\cos 2B + \cos(A-B) - \cos(A+B)\right]^2$$

$$= 1 - \frac{1}{2}\cos 2A - \frac{1}{2}\cos 2B + \cos(A-B) - \cos(A+B) - \frac{1}{2}\cos 2A + \frac{1}{4}\cos^2 2A + \frac{1}{4}\cos 2A \cos 2B$$
$$- \frac{1}{2}\cos 2A \cos(A-B) + \frac{1}{2}\cos 2A \cos(A+B) - \frac{1}{2}\cos 2B + \frac{1}{4}\cos 2A \cos 2B + \frac{1}{4}\cos^2 2B$$
$$- \frac{1}{2}\cos 2B \cos(A-B) + \frac{1}{2}\cos 2B \cos(A+B) + \cos(A-B) - \frac{1}{2}\cos 2A \cos(A-B) - \frac{1}{2}\cos 2B \cos(A-B)$$
$$+ \cos^2(A-B) - \cos(A-B)\cos(A+B) - \cos(A+B) + \frac{1}{2}\cos 2A \cos(A+B) + \frac{1}{2}\cos 2B \cos(A+B)$$
$$- \cos(A+B)\cos(A-B) + \cos^2(A+B)$$

$$y = x^4 = 1 - \cos 2A - \cos 2B - 2\cos(A+B) + 2\cos(A-B) + \frac{1}{4}\cos^2 2A + \frac{1}{4}\cos^2 2B + \frac{1}{2}\cos 2A \cos 2B$$
$$- \cos 2A \cos(A-B) + \cos 2A \cos(A+B) - \cos 2B \cos(A-B) + \cos 2B \cos(A+B)$$
$$- \cos(A-B)\cos(A+B) - \cos(A+B)\cos(A-B) + \cos^2(A+B) + \cos^2(A-B)$$

$$= 1 - \cos 2A - \cos 2B - 2\cos(A+B) + 2\cos(A-B) + \frac{1}{8} + \frac{1}{8}\cos 4A + \frac{1}{8} + \frac{1}{8}\cos 4B + \frac{1}{4}\cos 2(A+B)$$
$$+ \frac{1}{4}\cos 2(A-B) - \frac{1}{2}[\cos(3A-B) + \cos(A+B)] + \frac{1}{2}[\cos(3A+B) + \cos(A-B)]$$
$$- \frac{1}{2}[\cos(A+B) + \cos(3B-A)] + \frac{1}{2}[\cos(3B+A) + \cos(B-A)] - [\cos 2A + \cos(-2B)] + \frac{1}{2}$$
$$+ \frac{1}{2}\cos 2(A+B) + \frac{1}{2} + \frac{1}{2}\cos 2(A-B)$$

$$= \frac{9}{4} - 2\cos 2A - 2\cos 2B - 3\cos(A+B) + 3\cos(A-B) + \frac{1}{8}\cos 4A + \frac{1}{8}\cos 4B + \frac{3}{4}\cos 2(A+B)$$
$$+ \frac{3}{4}\cos 2(A-B) - \frac{1}{2}\cos(3A-B) + \frac{1}{2}\cos(3A+B) - \frac{1}{2}\cos(3B-A) + \frac{1}{2}\cos(3B+A)$$

$$\%THD = \left[\frac{\left(\frac{2}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2 + \left(\frac{1}{8\sqrt{2}}\right)^2 + \left(\frac{1}{8\sqrt{2}}\right)^2}{\left(\frac{2}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2 + \left(\frac{1}{8\sqrt{2}}\right)^2 + \left(\frac{1}{8\sqrt{2}}\right)^2} \right]^{1/2} \times 100$$

=100%

For a 5th Order Non-linear System (with 2 sinusoids)

$$y = x^5 \quad = (\sin A + \sin B)^5$$

$$y = x^4 x = (\sin A + \sin B)^4 (\sin A + \sin B)$$

$$= \begin{bmatrix} \frac{9}{4} + 2\cos 2A - 2\cos 2B - 3\cos(A+B) + 3\cos(A-B) + \frac{1}{8}\cos 4A + \frac{1}{8}\cos 4B \\ + \frac{3}{4}\cos 2(A+B) + \frac{3}{4}\cos 2(A-B) - \frac{1}{2}\cos(3A-B) + \frac{1}{2}\cos(3A+B) \\ - \frac{1}{2}\cos(3B-A) + \frac{1}{2}\cos(3B+A) \end{bmatrix} (\sin A + \sin B)$$

$$= \frac{9}{4}\sin A + \frac{9}{4}\sin B + 2\sin A\cos 2A + 2\sin B\cos 2A - 2\sin A\cos 2B - 2\sin B\cos 2B$$

$$-3\sin A\cos(A+B)-3\sin B\cos(A+B)+3\sin A\cos(A-B)+3\sin B\cos(A-B)$$
$$+\frac{1}{8}\sin A\cos 4A+\frac{1}{8}\sin B\cos 4A+\frac{1}{8}\sin A\cos 4B+\frac{1}{8}\sin B\cos 4B+\frac{3}{4}\sin A\cos 2(A+B)$$
$$+\frac{3}{4}\sin B\cos 2(A+B)+\frac{3}{4}\sin A\cos 2(A-B)+\frac{3}{4}\sin B\cos 2(A-B)-\frac{1}{2}\sin A\cos(3A-B)$$
$$-\frac{1}{2}\sin B\cos(3A-B)+\frac{1}{2}\sin A\cos(3A+B)+\frac{1}{2}\sin B\cos(3A+B)-\frac{1}{2}\sin A\cos(3B-A)$$
$$-\frac{1}{2}\sin B\cos(3B-A)+\frac{1}{2}\sin A\cos(3B+A)+\frac{1}{2}\sin B\cos(3B+A)$$

$$y=\frac{9}{4}\sin A+\frac{9}{4}\sin B+\sin 3A-\sin A+\sin(2A+B)-\sin(2A-B)+\sin(A+2B)+\sin(A-2B)$$

$$+\sin 3B-\sin B-\frac{3}{2}\sin(2A+B)+\frac{3}{2}\sin B-\frac{3}{2}\sin(A+2B)+\frac{3}{2}\sin A+\frac{3}{2}\sin(2A-B)+\frac{3}{2}\sin B$$
$$+\frac{3}{2}\sin A-\frac{3}{2}\sin(A-2B)+\frac{1}{16}\sin 5A-\frac{1}{16}\sin 3A+\frac{1}{16}\sin(4A+B)-\frac{1}{16}\sin(4A-B)+\frac{1}{16}\sin(A+4B)$$
$$+\frac{1}{16}\sin(A-4B)+\frac{1}{16}\sin 5B-\frac{1}{16}\sin 3B+\frac{3}{8}\sin(3A+2B)-\frac{3}{8}\sin(A+2B)+\frac{3}{8}\sin(2A+3B)$$
$$-\frac{3}{8}\sin(2A+B)+\frac{3}{8}\sin(3A-2B)-\frac{3}{8}\sin(A-2B)+\frac{3}{8}\sin(2A-B)-\frac{3}{8}\sin(2A-3B)-\frac{1}{4}\sin(4A-B)$$
$$+\frac{1}{4}\sin(2A-B)-\frac{1}{4}\sin 3A+\frac{1}{4}\sin(3A+2B)+\frac{1}{4}\sin(4A+B)-\frac{1}{4}\sin(2A+B)+\frac{1}{4}\sin(3A+2B)$$
$$-\frac{1}{4}\sin 3A-\frac{1}{4}\sin 3B-\frac{1}{4}\sin(2A-3B)-\frac{1}{4}\sin 4B-\frac{1}{4}\sin(A-2B)+\frac{1}{4}\sin(2A+3B)-\frac{1}{4}\sin 3B$$
$$+\frac{1}{4}\sin(A+4B)-\frac{1}{4}\sin(A+2B)$$

$$
\begin{aligned}
y = {} & \frac{5}{4}\sin A+\frac{5}{4}\sin B+\frac{3}{2}\sin A+\frac{3}{2}\sin B+\frac{3}{2}\sin B+\frac{3}{2}\sin A+\sin 3A-\frac{1}{16}\sin 3A-\frac{1}{4}\sin 3A-\frac{1}{4}\sin 3A \\
& +\sin 3B-\frac{1}{16}\sin 3B-\frac{1}{4}\sin 3B-\frac{1}{4}\sin 3B+\sin(2A+B)-\frac{3}{2}\sin(2A+B)-\frac{3}{8}\sin(2A+B) \\
& -\frac{1}{4}\sin(2A+B)-\sin(2A-B)+\frac{3}{2}\sin(2A-B)+\frac{3}{8}\sin(2A-B)+\frac{1}{4}\sin(2A-B)+\sin(A+2B) \\
& -\frac{3}{2}\sin(A+2B)-\frac{3}{8}\sin(A+2B)-\frac{1}{4}\sin(A+2B)+\sin(A-2B)-\frac{3}{2}\sin(A-2B)-\frac{3}{8}\sin(A-2B) \\
& -\frac{1}{4}\sin(A-2B)+\frac{1}{16}\sin(A+4B)+\frac{1}{4}\sin(A+4B)+\frac{1}{16}\sin(A-4B)-\frac{3}{8}\sin(2A-3B)-\frac{1}{4}\sin(2A-3B) \\
& +\frac{3}{8}\sin(2A+3B)+\frac{1}{4}\sin(2A+3B)+\frac{3}{8}\sin(3A+2B)+\frac{1}{4}\sin(3A+2B)+\frac{1}{4}\sin(3A+2B) \\
& +\frac{1}{16}\sin(4A+B)+\frac{1}{4}\sin(4A+B)-\frac{1}{16}\sin(4A-B)-\frac{1}{4}\sin(4A-B)+\frac{3}{8}\sin(3A-2B)-\frac{1}{4}\sin 4B \\
& +\frac{1}{16}\sin 5A+\frac{1}{16}\sin 5B
\end{aligned}
$$

$$
\begin{aligned}
y = {} & \frac{17}{4}\sin A+\frac{17}{4}\sin B+\frac{7}{16}\sin 3A+\frac{7}{16}\sin 3B-\frac{9}{8}\sin(2A+B)+\frac{9}{8}\sin(2A-B)-\frac{9}{8}\sin(A+2B) \\
& -\frac{9}{8}(A-2B)+\frac{5}{16}\sin(A+4B)+\frac{1}{16}\sin(A-4B)-\frac{5}{8}\sin(2A-3B)+\frac{5}{8}\sin(2A+3B) \\
& +\frac{7}{8}\sin(3A+2B)+\frac{8}{16}\sin(4A+B)-\frac{5}{16}\sin(4A-B)+\frac{3}{8}\sin(3A-2B)-\frac{1}{4}\sin 4B+\frac{1}{16}\sin 5A \\
& +\frac{1}{16}\sin 5B
\end{aligned}
$$

Appendix B

Frequency Normalization

Let $x_1(t) = \sin 2\pi f_1 t$ (B1)

If $x_1(t)$ is sampled at a rate, T, such that

$$T = \frac{1}{f_s} \tag{B2}$$

where f_s is the sampling rate, then the resulting sample $x(t)$ becomes,

$$x(nT) = \sin 2\pi f_1 nT$$

$$= \sin 2\pi \frac{f_1}{f_s} n \stackrel{\Delta}{\equiv} x(n) \tag{B3}$$

where the normalized frequency

$$f_n = \frac{f_1}{f_s}, \text{ or}$$

$$x(n) = 2\pi f_n n \tag{B4}$$

Appendix C

Derivation of Spectral Centroid

Reducing a spectral distribution to an impulse centered at the centroid of the distribution

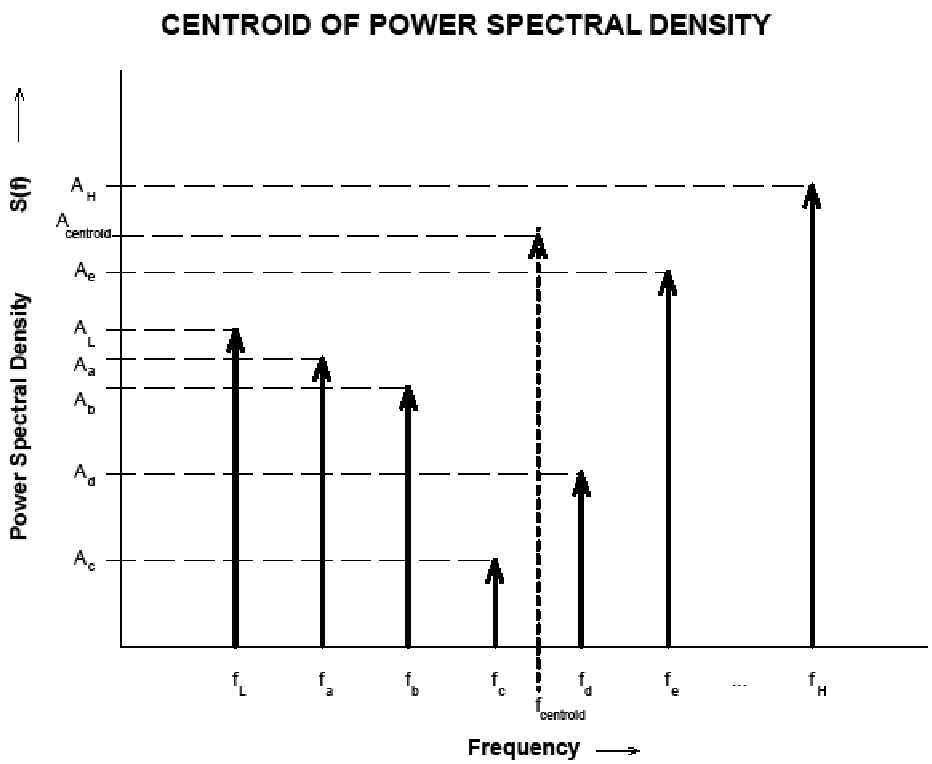

Fig. 18 Centroid of Power Spectral Density

Total Power Between $f_L < f < f_H$

$$\int_{f_L}^{f_H} \begin{bmatrix} A_L\delta(f-f_L)+A_a\delta(f-f_a)+A_b\delta(f-f_b)+A_c\delta(f-f_c) \\ +A_d\delta(f-f_d)+A_e\delta(f-f_e)+....A_H\delta(f-f_H) \end{bmatrix} df$$

$$= A_L + A_a + A_b + A_c + A_d + A_e + ...A_H \qquad \text{(C1)}$$

$$\int_{f_L}^{f_H} A_{CENTROID}\,\delta\left(f - f_{CENTROID}\right)df = A_{CENTROID} \tag{C2}$$

Where $A_{CENTROID} = A_L + A_a + A_b + A_c + A_d + A_e + \ldots A_H$ (C3)

The spectral centroid may be defined as,

$$\int_{-\infty}^{\infty}\left(f - f_{CENTROID}\right)S\left(f\right)df = 0 \tag{C4}$$

$$= \int_{-\infty}^{\infty}\left(f - f_{CENTROID}\right)\begin{bmatrix} A_L\delta\left(f - f_L\right) + A_a\delta\left(f - f_a\right) + A_b\delta\left(f - f_b\right) + A_c\delta\left(f - f_c\right) \\ + A_d\delta\left(f - f_d\right) + A_e\delta\left(f - f_e\right) + \ldots A_H\delta\left(f - f_H\right) \end{bmatrix}df = 0$$

$$= A_L\left(f_L - f_{CENTROID}\right) + A_a\left(f_a - f_{CENTROID}\right) + A_b\left(f_b - f_{CENTROID}\right) + A_c\left(f_c - f_{CENTROID}\right)$$
$$+ A_d\left(f_d - f_{CENTROID}\right) + A_e\left(f_e - f_{CENTROID}\right) + \ldots A_H\left(f_H - f_{CENTROID}\right) = 0$$

Solving Eq. C3 for $f_{CENTROID}$,

$$f_{CENTROID} = \frac{A_L f_L + A_a f_a + A_b f_b + A_c f_c + A_d f_d + A_e f_e + A_H f_H}{A_L + A_a + A_b + A_c + A_d + A_e + A_H} \tag{C5}$$

Where $A_{CENTROID} = A_L + A_a + A_b + A_c + A_d + A_e + \ldots A_H$

$$\therefore f_{CENTROID} = \frac{A_L}{A_{CENTROID}} f_L + \frac{A_a}{A_{CENTROID}} f_a + \frac{A_b}{A_{CENTROID}} f_b$$
$$+ \frac{A_c}{A_{CENTROID}} f_c + \frac{A_d}{A_{CENTROID}} f_d + \frac{A_e}{A_{CENTROID}} f_e + \ldots \frac{A_H}{A_{CENTROID}} f_H \tag{C6}$$

Appendix D

THD Program MATLAB Code


```
function [thdpercent] = compthdpercent21()
%   compthdpercent21 is a function which computes the total harmonic distortion
%   in percent for a given time invariant system.  This function prompts
%   the user for the following: NPTS (the number of sample points), the sampling
%   frequency, the number of sinusoidal vectors, the frequency and peak
%   amplitude for each vector, the order of the nonlinearity.

% Menu System for User Input

fprintf('NPTS = 2^z\n')
z=input('Enter z: ')
NPTS=2^z
n=1:1:NPTS;
fs=input('Enter the sampling frequency (Hz): ')
numbervectors=input('Enter number of sinusoidal vectors: ')
fminbound=10*fs/NPTS;
fmaxbound=fs/2;
for i=1:1:numbervectors
    fprintf('For vector %.1g\n', i)
    fprintf('The frequency (Hz) must be greater than %.2f (Hz)', fminbound)
    fprintf(' but less than %.2f (Hz)\n', fmaxbound)
    freq(i)=input('Enter frequency (Hz): ')
    amp(i)=input('Enter peak amplitude: ')
end
fmin=min(freq);
deltaf=fmin/200;
NPTSlowerbound=fs/deltaf;
NPTSupperbound=8192;
while (NPTS <= NPTSlowerbound) & (NPTS <= NPTSupperbound)
    z=z+1
    NPTS=2^z
    n=1:1:NPTS;
    fprintf('NPTS has been changed to %6g\n', NPTS)
end
fprintf('The following prompts will define the coefficients\n')
fprintf('of the polynomial representing the system nonlinearity\n')
b(1)=input('Enter a scaling factor between 0 and 1 for the DC component: ')
b(2)=input('Enter a scaling factor between 0 and 1 for the 1st order component: ')
b(3)=input('Enter a scaling factor between 0 and 1 for the 2nd order component: ')
b(4)=input('Enter a scaling factor between 0 and 1 for the 3rd order component: ')
b(5)=input('Enter a scaling factor between 0 and 1 for the 4th order component: ')
b(6)=input('Enter a scaling factor between 0 and 1 for the 5th order component: ')
order=find(b,1,'last')-1;

% Formation of Array(Matrix) of Sinusoidal Vector(s)
for i=1:1:numbervectors
    x(i,:)=amp(i)*sin(2*pi*(freq(i)/fs)*(n-1));
end

% Formation of Composite 1-D Input Signal if it doesn't already exist
if (size(x,1)>1)
```

```
    xsum=sum(x);
    x=xsum;
end

% Compute PSD on input signal
NPTS
X=(fft(x))/NPTS;
Sx=(abs(X)).^2;
Sxpos=Sx(1:1:NPTS/2 + 1);
Sxposfreq=0:fs/NPTS:(fs*(NPTS/2))/NPTS;
h1=figure;
plot(Sxposfreq,Sx(1:NPTS/2+1)),title('Input Power Spectral Density vs. Frequency')

% Identify Frequencies with maximum amplitude for input PSD

for i=1:1:numbervectors
    maxampfreqin(i)=find(Sxpos==max(Sxpos));
    maxampfreqin(i)=maxampfreqin(i)-1;
    Sxpos(1,maxampfreqin(i)+1)=0;
end
maxampfreqin=sort(maxampfreqin)

% Identify fundamental frequencies for input PSD

for i=1:1:size(maxampfreqin,2)
    if maxampfreqin(i) - 1 ~=0
        Sxfund(i)=maxampfreqin(i);
    end
end
format bank
Sxfund=Sxfund*(fs/NPTS)
format short

% Adaptive Process to compute mean and variance for a shrinking window

if (size(maxampfreqin,2))==1
    freqwindowleft=zeros(1,maxampfreqin(1)-1);
    freqwindowright=zeros(1,maxampfreqin(1)-1);
    M=zeros(2,NPTS/2+1);
    V=zeros(2,NPTS/2);
    freqmidpoint=mean(1:maxampfreqin);
    freqwindowleft(1:maxampfreqin)=1:1:maxampfreqin;
    freqwindowright(1:maxampfreqin)=maxampfreqin:-1:1;
    for j=1:1:maxampfreqin
        if freqwindowleft(j) <= freqmidpoint
            M(1,j)=mean(Sx(freqwindowleft(j):1:freqwindowright(j)));
            V(1,j)=var(Sx(freqwindowleft(j):1:freqwindowright(j)));
        end
    end
    for j=maxampfreqin:1:(NPTS/2+1)
        M(2,j)=mean(Sx((j):1:NPTS/2+1));
```

```
    end
    for j=maxampfreqin:1:(NPTS/2+1)
        V(2,j)=var(Sx((j):1:NPTS/2+1));
    end
    Vderivative(1,(1:floor(freqmidpoint)-1))=diff(V(1,(1:floor(freqmidpoint))));
    Vderivative(2,(maxampfreqin:NPTS/2))=diff(V(2,(maxampfreqin:NPTS/2+1)));
    Vderivativeabs=abs(Vderivative);
    [Vderivativeabsmaxamp(1,1),Vderivativeabsmaxamp(1,2)]=max(Vderivativeabs(1,:));
    [Vderivativeabsmaxamp(2,1),Vderivativeabsmaxamp(2,2)]=max(Vderivativeabs(2,:));
    for i=1:1:2
        temp=find(Vderivativeabs(i,Vderivativeabsmaxamp(i,2):NPTS/2)<=(Vderivativeabsmaxamp(i,1)*0.01));
        Vderivativeabsfreqmin(i,1)=min(temp);
        if Vderivativeabsmaxamp(i,2)-1==0
        else
            Vderivativeabsfreqmin(i,1)=(Vderivativeabsfreqmin(i,1)+Vderivativeabsmaxamp(i,2)-1);
        end
        Inputnoisefloortemp(i,1)=M(i,(Vderivativeabsfreqmin(i,1)));
    end
    Inputnoisefloor=max(Inputnoisefloortemp);
else
    if maxampfreqin(1)~=1
        maxampfreqintemp(1)=1;
        for i=1:1:size(maxampfreqin,2)
            maxampfreqintemp(i+1)=maxampfreqin(i);
        end
        maxampfreqintemp;
    end
    numberofwindows=size(maxampfreqintemp,2)-1;
    freqwindowleft=zeros(numberofwindows,NPTS/2+1);
    freqwindowright=zeros(numberofwindows,NPTS/2+1);
    M=zeros(numberofwindows,NPTS/2+1);
    V=zeros(numberofwindows,NPTS/2+1);
    for i=1:1:numberofwindows
        freqmidpoint(i)=mean(maxampfreqintemp(i:i+1));
        freqwindowleft(i,maxampfreqintemp(i):1:maxampfreqintemp(i+1))=maxampfreqintemp(i):1:maxampfreqintemp(i+1);
        freqwindowright(i,maxampfreqintemp(i):1:maxampfreqintemp(i+1))=maxampfreqintemp(i+1):-1:maxampfreqintemp(i);
        for j=find(freqwindowleft(i,:),1):1:find(freqwindowleft(i,:),1)+nnz(freqwindowleft(i,:))-1
            if freqwindowleft(i,j) <= freqmidpoint(i)
                M(i,j)=mean(Sx(freqwindowleft(i,j):1:freqwindowright(i,j)));
                V(i,j)=var(Sx(freqwindowleft(i,j):1:freqwindowright(i,j)));
            end
        end
    end
    freqmidpoint
    for j=maxampfreqintemp(size(maxampfreqintemp,2)):1:(NPTS/2+1)
        M(numberofwindows+1,j)=mean(Sx((j):1:NPTS/2+1));
```

```
        V(numberofwindows+1,j)=var(Sx((j):1:NPTS/2+1));
    end
    for i=1:1:numberofwindows
        Vderivative(i,(maxampfreqintemp(i):floor(freqmidpoint(i))-1))=diff(V(i,(maxampfreqintemp(i):floor(freqmidpoint(i)))));
    end
    Vderivative(numberofwindows+1,(maxampfreqintemp(size(maxampfreqintemp,2)):NPTS/2))=diff(V(numberofwindows+1,(maxampfreqintemp(size(maxampfreqintemp,2)):NPTS/2+1)));
    Vderivativeabs=abs(Vderivative);
    for i=1:1:numberofwindows+1
        [Vderivativeabsmaxamp(i,1),Vderivativeabsmaxamp(i,2)]=max(Vderivativeabs(i,:));
    end
    for i=1:1:numberofwindows+1
        temp=find(Vderivativeabs(i,Vderivativeabsmaxamp(i,2):NPTS/2)<=(Vderivativeabsmaxamp(i,1)*0.01));
        Vderivativeabsfreqmin(i,1)=min(temp);
        if Vderivativeabsmaxamp(i,2)-1==0
        else
            Vderivativeabsfreqmin(i,1)=(Vderivativeabsfreqmin(i,1)+Vderivativeabsmaxamp(i,2)-1);
        end
        Inputnoisefloortemp(i,1)=M(i,(Vderivativeabsfreqmin(i,1)));
    end
    Inputnoisefloor=max(Inputnoisefloortemp);
end

% Generating system nonlinear model

y=b(1) + b(2).*x + b(3).*x.^2 + b(4).*x.^3 + b(5).*x.^4 + b(6).*x.^5;
h2=figure;
plot(x,y),title('System Nonlinearity')
Y=(fft(y))/NPTS;
Sy=(abs(Y)).^2;
Sypos=Sy(1:1:NPTS/2 + 1);
Syposfreq=Sxposfreq;
h3=figure;
plot(Syposfreq,Sy(1:NPTS/2+1)),title('Output Power Spectral Density vs. Frequency')

% Identify Frequencies with maximum amplitude for output PSD

Outputnoisefloor=Inputnoisefloor;
maxampfreqout=find(Sypos>Outputnoisefloor)

% Identify Frequencies too close together and convert to a single frequency

for i=1:1:size(maxampfreqout,2)
    amplitude(i)=Sy(maxampfreqout(i));
end
freqout=1;
```

```
toconsolidatefreq(freqout,1)=maxampfreqout(1);
toconsolidateamp(freqout,1)=amplitude(1);
for i=1:1:size(maxampfreqout,2)-1
    if (maxampfreqout(i+1) - maxampfreqout(i) == 1)
        toconsolidatefreq(freqout,i+1)=maxampfreqout(i+1);
        toconsolidateamp(freqout,i+1)=amplitude(i+1);
    else
        freqout=freqout+1;
        toconsolidatefreq(freqout,i+1)=maxampfreqout(i+1);
        toconsolidateamp(freqout,i+1)=amplitude(i+1);
    end
end
S=sparse(toconsolidatefreq);
R=spones(S);
S=S-R;                              % Leaving matlab array index domain
toconsolidatefreq=full(S);          % and entering sample frequency domain
if toconsolidatefreq(1,:) == 0
    toconsolidatefreq(:,1)=[];
    toconsolidatefreq(1,:)=[];
    toconsolidateamp(:,1)=[];
    toconsolidateamp(1,:)=[];
end
toconsolidatefreq
toconsolidateamp
freqout=size(toconsolidatefreq,1)
for i=1:1:freqout
    width(i)=size(find(toconsolidatefreq(i,:)),2);
end
width=transpose(width)
for i=1:1:freqout
    consolidateamp(i,1)=sum(toconsolidateamp(i,:));
end
consolidateamp
consolidatefreq=zeros(freqout,1);
counter=1;
for i=1:1:freqout
    for j=counter:1:counter + width(i,1)-1
        consolidatefreq(i,1)=consolidatefreq(i,1) + toconsolidateamp(i,j)
*toconsolidatefreq(i,j);
    end
    counter=counter + width(i,1);
end
consolidatefreq
consolidatefreq=consolidatefreq./consolidateamp;
format bank
calmaxampfreqout=consolidatefreq*(fs/NPTS)
format short

% Identify if fundamental and first 10 harmonics are present in output PSD

calmaxampfreqout=transpose(calmaxampfreqout);
```

```
harmonics=[1 2 3 4 5 6 7 8 9 10];
offset=0.02;
harmonicsouttemp=zeros(size(Sxfund,2),size(calmaxampfreqout,2));
for i=1:1:size(Sxfund,2)
    for j=1:1:size(calmaxampfreqout,2)
        harmonicsout(i,j)=calmaxampfreqout(j)/Sxfund(i);
        for k=1:1:size(harmonics,2)
            if ((harmonicsout(i,j) >= harmonics(k) - offset) & (harmonicsout(i,j) <= harmonics(k) + offset))
                harmonicsouttemp(i,j)=harmonicsout(i,j);
            end
        end
    end
end
harmonicsouttemp=round(harmonicsouttemp);
for i=1:1:size(harmonicsouttemp,2)
    if nnz(harmonicsouttemp(:,i))>1
        fprintf('Two or more input frequencies are multiples of each other\n')
        fprintf('Please try again\n')
        return
    end
end

% Decode fundamental and harmonics present in (harmonicsouttemp) output PSD
% to begin to sum harmonic powers

harmonicpower=0;
fundamentalpower=0;
[indexi,indexj]=find(harmonicsouttemp)
if size(indexj,1) == 1
    index=2;
else
    index=1;
end
for k=1:1:size(indexj,index)
    if harmonicsouttemp(indexi(k),indexj(k)) > 1
        harmonicpower=harmonicpower + (consolidateamp(indexj(k))*2);
    elseif harmonicsouttemp(indexi(k),indexj(k)) == 1
        fundamentalpower=fundamentalpower + (consolidateamp(indexj(k))*2);
    end
end
harmonicpower
fundamentalpower
totalpower=fundamentalpower + harmonicpower
THD=sqrt(harmonicpower/totalpower)*100
thislinemeansnothing=5;
```

Vita

Lawrence J. Tedesco, Jr. was born in Metairie, LA and received his B.S. from Tulane University in Biomedical Engineering in 1998. Mr. Tedesco is Chief Biomedical Design Engineer and Vice–President of Biomedtronics, L.L.C. since 2000.